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The Stability of a Generalized Radical Reciprocal Quadratic Functional Equation in Felbin's Space

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Abstract

In this paper, the authors introduce a new generalized radical reciprocal quadratic functional equation of the form

$$f\left(\frac{\sqrt{x^2+y^2}}{n}\right) \pm m f\left(\sqrt{x^2+y^2}\right) = (n^2 \pm m)\frac{f(x)f(y)}{f(x)+f(y)}$$

and investigate its generalized Hyers-Ulam stability in Felbin's type fuzzy normed linear spaces. Also we give counter example for the Hyers-Ulam-Rassias stability of the radical reciprocal quadratic functional equation for some cases.

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Key words: Reciprocal functional equations, Felbin's Space, Generalized Hyers-Ulam stability.

1 Introduction

The stability problem of functional equations originates from the fundamental question: When is it true that a mathematical object satisfying a certain property approximately must be close to an object satisfying the property exactly?

In connection with the above question, in 1940, S.M. Ulam [36] raised a question concerning the stability of homomorphisms. Let G be a group and let G' be a metric group with $d(.,.)$. Given $\epsilon > 0$ does there exist a $\delta > 0$ such that if a function $f : G \to G'$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all $x, y \in G$, then there is a homomorphism $H : G \to G'$ with $d(f(x), H(x)) < \epsilon$ for all $x \in G$?

The first partial solution to Ulam's question was given by D.H. Hyers [9]. He considered the case of approximately additive mappings $f : E \to E'$ where E and E' are Banach spaces and f satisfies Hyers inequality

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon$$

for all $x, y \in E$, it was shown that the limit $a(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ exists for all $x \in E$ and that $a : E \to E'$ is the unique additive mapping satisfying

$$\|f(x) - a(x)\| \leq \epsilon.$$

Moreover, it was proved that if $f(tx)$ is continuous in t for each fixed $x \in E$, then a is linear. In this case, the Cauchy additive functional equation $f(x+y) = f(x) + f(y)$ is said to satisfy the Hyers-Ulam stability.

In 1978, Th.M. Rassias [35] provided a generalized version of the theorem of Hyers which permitted the Cauchy difference to become unbounded. He proved the following theorem.

Theorem 1. [**Th.M. Rassias**] *If a function $f : E \to E'$ between Banach spaces satisfies the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \theta \left(|x\|^p + \|y\|^p\right) \tag{1}$$

for some $\theta \geq 0$, $0 \leq p < 1$ and for all $x, y \in E$, then there exists a unique additive function $a : E \to E'$ such that

$$\|f(x) - a(x)\| \leq \frac{2\theta}{2 - 2^p} \|x\|^p \tag{2}$$

for all $x \in E$. Moreover, if $f(tx)$ is continuous in t for each fixed $x \in E$, then a is linear.